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4–5 Factoring Polynomials

Objective: To factor polynomials by using the GCF, by recognizing special products, and by grouping terms.

Vocabulary

Factor a polynomial To express a polynomial as a product of other polynomials.

Greatest monomial factor The GCF of the terms of a polynomial.

Special factoring patterns

Perfect square trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $\quad a^2 - 2ab + b^2 = (a - b)^2$
Difference of squares	$a^2 - b^2 = (a + b)(a - b)$
Sum of cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 1 Factor: **a.** $3x^4 - 6x^3 + 12x^2$ **b.** $8a^3b - 12a^2b^2$

Solution **a.** $3x^4 - 6x^3 + 12x^2 = 3x^2(x^2 - 2x + 4)$ ← The GCF of the terms is $3x^2$.

b. $8a^3b - 12a^2b^2 = 4a^2b(2a - 3b)$ ← The GCF of the terms is $4a^2b$.

Factor each polynomial.

1. $y^2 + y$ **$y(y + 1)$**
2. $4x - 28$ **$4(x - 7)$**
3. $8a^4 - 14a^2b$ **$2a^2(4a^2 - 7b)$**
4. $6x^2 - 8x^3 - 10x^4$ **$2x^2(3 - 4x - 5x^2)$**
5. $10x^8 + 15x^7 - 35x^5$ **$5x^5(2x^3 + 3x^2 - 7)$**
6. $11a^3b - 22a^2b^2 + 55ab^3$ **$11ab(a^2 - 2ab + 5b^2)$**

Example 2 Factor: **a.** $z^2 + 8z + 16$ **b.** $9x^2 - 6xy + y^2$ **c.** $36m^2 - 49n^2$

Solution **a.** $z^2 + 8z + 16 = z^2 + 2(z)(4) + (4)^2$ ← perfect square trinomial
$= (z + 4)^2$

b. $9x^2 - 6xy + y^2 = (3x)^2 - 2(3x)(y) + y^2$ ← perfect square trinomial
$= (3x - y)^2$

c. $36m^2 - 49n^2 = (6m)^2 - (7n)^2$ ← difference of squares
$= (6m + 7n)(6m - 7n)$

Example 3 Factor $2a^5 - 162a$.

Solution Always begin by looking for the GCF of the terms. If the GCF is not 1, then factor the GCF out. The GCF of $2a^5$ and $-162a$ is $2a$.

$2a^5 - 162a = 2a(a^4 - 81)$
$= 2a[(a^2)^2 - (9)^2]$ ← difference of squares
$= 2a(a^2 + 9)(a^2 - 9)$
$= 2a(a^2 + 9)(a^2 - 3^2)$ ← difference of squares
$= 2a(a^2 + 9)(a + 3)(a - 3)$

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4–5 Factoring Polynomials (continued)

14. $(3m + 8)(3m - 8)$ **15.** $(5x + 2y)^2$ **16.** $(9p + 7q)(9p - 7q)$ **18.** $5c(c + 3d)^2$

Factor each polynomial.

20. $4y(x + 3)(x - 3)$ **21.** $(4n^2 + 1)(2n + 1)(2n - 1)$

7. $x^2 + 10x + 25$ **$(x + 5)^2$**
8. $a^2 - 16a + 64$ **$(a - 8)^2$**
9. $4y^2 - 12y + 9$ **$(2y - 3)^2$**
10. $4b^2 + 28b + 49$ **$(2b + 7)^2$**
11. $x^2 - 16$ **$(x + 4)(x - 4)$**
12. $y^2 - 100$ **$(y + 10)(y - 10)$**
13. $4k^2 - 25$ **$(2k + 5)(2k - 5)$**
14. $9m^2 - 64$
15. $25x^2 + 20xy + 4y^2$
16. $81p^2 - 49q^2$
17. $3x^2 + 12x + 12$ **$3(x + 2)^2$**
18. $5c^3 + 30c^2d + 45cd^2$
19. $rt^2 - r$ **$r(t + 1)(t - 1)$**
20. $4x^2y - 36y$
21. $16n^4 - 1$

Example 4 Factor: **a.** $a^3 - 8$ **b.** $27x^3 + 1$

Solution **a.** $a^3 - 8 = a^3 - 2^3 = (a - 2)(a^2 + 2a + 4)$ ← difference of cubes

b. $27x^3 + 1 = (3x)^3 + 1^3 = (3x + 1)(9x^2 - 3x + 1)$ ← sum of cubes

Factor each polynomial.

22. $(x + 1)(x^2 - x + 1)$ **23.** $(4 - a)(16 + 4a + a^2)$ **24.** $(t + 5)(t^2 - 5t + 25)$ **25.** $(10c - 3)(100c^2 + 30c + 9)$

22. $x^3 + 1$
23. $64 - a^3$
24. $t^3 + 125$
25. $1000c^3 - 27$

Example 5 Factor: **a.** $2a^3 - 3a^2 - 4a + 6$ **b.** $12x^3 + 4x^2y - 3x - y$

Solution **a.** The first and second terms have a common factor of a^2, and the third and fourth terms have a common factor of -2. Factor by grouping terms.

$2a^3 - 3a^2 - 4a + 6 = (2a^3 - 3a^2) + (-4a + 6)$
$= a^2(2a - 3) - 2(2a - 3)$ Common factor is $2a - 3$.
$= (a^2 - 2)(2a - 3)$ Factor out $(2a - 3)$.

b. The first and third terms have a common factor of $3x$, and the second and fourth terms have a common factor of y. Factor by grouping terms.

$12x^3 + 4x^2y - 3x - y = (12x^3 - 3x) + (4x^2y - y)$
$= 3x(4x^2 - 1) + y(4x^2 - 1)$
$= (3x + y)(4x^2 - 1)$ ← difference of squares
$= (3x + y)(2x + 1)(2x - 1)$

Factor each polynomial. **26.** $(a - 3)(b + 2)$ $(m + 1)(n - 2)$ $(5a^2 + 2)(4a - 1)$

26. $a(b + 2) - 3(b + 2)$
27. $m(n - 2) - (2 - n)$
28. $20a^3 - 5a^2 + 8a - 2$
29. $10y^3 + 10y^2 + 3y + 3$ **$(10y^2 + 3)(y + 1)$**
30. $9a^2b - 8a^2 - 9b + 8$ **$(9b - 8)(a + 1)(a - 1)$**
31. $5x^2y - 7x^2 - 7 + 5y$ **$(x^2 + 1)(5y - 7)$**

Mixed Review Exercises

Write as a simplified polynomial. **$12a^2 + a - 6$** **$4r^3s^2 - 5r^2s^3$**

1. $(a + 2)^2$ **$a^2 + 4a + 4$**
2. $(3a - 2)(4a + 3)$
3. $r^2s^2(4r - 5s)$
4. $(5a - 2) - (3 - a)$ **$6a - 5$**
5. $(x^2 - 2)(x + 5)$ **$x^3 + 5x^2 - 2x - 10$**
6. $(-a)^2(2a^2)^3$ **$8a^8$**

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